Comments on Chapter 5 of G. I. Shipov's "A Theory of Physical Vacuum". Part II

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Abstract—**This paper, part II of the series, discusses mathematical problems and inconsistencies in the description of equations of motion of spinning matter and in the applications of the theory found in Ch. 5 of the monograph "A Theory of Physical Vacuum" by G. I. Shipov, as well as in the papers devoted to "Descartesian mechanics". In particular it is pointed out that in addition to calculational errors there are also mathematical contradictions of the fundamental nature in the very approach to the problem.**

Index Terms—**torsion, absolute parallelism, Einstein-Cartan theory, spin, geodesics, Mathisson-Papapetrou equations, 4D gyroscope**

I. Introduction

In Einstein's theory of gravitation, also known as General Relativity, the physical reality has a dual character: it is split into space (endowed with a Riemannian metric), and mass (usually represented by some kind of "field", and/or "energy-momentumm"). The idea of this theory is briefly summarized by the famous phrase attributed to John. A. Wheeler *Space tells matter how to move. Matter tells space how to curve.* Mathematically, the part "Space tells matter how to move", is expressed by geodesic equations: particles move in space along the shortest (equivalently: "as straight as possible") paths according to the spacetime metric.

Yet physics is not that simple. Matter, apart from its momentum-energy, has also another important property related to rotations: angular momentum and/or spin. Spinning particles do not necessarily follow the shortest lines. Spin can directly "feel" the curvature of space. The corresponding equations of motion are known as Mathisson-Papapetrou equations; they can be derived without specifying any particular "Lagrangians", from the basic principles of invariance alone [1].[1]

About the same time that physicists realized that particles can have spin (late 1920-ties), they also discovered another property that space can be endowed with: torsion. Elie Cartan speculated about the possible relation between spinning of matter and twisting of space, Albert Einstein tried to construct a unified field theory in spacetime admitting both: curvature and torsion [2]. Torsion is somewhat more difficult to visualize than curvature - it relates to "defects" that lead to nonclosure of parallelograms. A new idea appeared in physics that was worth of attention: spinning matter is the source of torsion, and torsion tells matter how to spin. This ideawas closely related to the old puzzle of inertia that General Relativity was not able to answer to everybody's satisfaction: how do gyroscopes know which direction to keep? If everything is relative, what is the meaning of "spinning"? Spinning with respect to what?

Mathisson-Papapetrou equations have been extended so as to include the interaction of spin (or angular momentum) with torsion [3], [4].[2] Their derivation relies on the conservation laws alone, and these, on the other hand, are direct consequences of the general invariance and gauge invariance principles. Yet not all physicists agree that this is the only way to describe the motion of gyroscopes in spacetime with torsion. Some physicists still argue that physics should follow the way that is math-

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[1] The Mathisson-Papapetrou equations alone are not deterministic. To have a deterministic set, the internal structure of the particle must be specified. It can be done, for instance, by setting some relations between the spin and the momentum or velocity.

[2] The paper by S. Sternberg uses mathematics that very few physicists can swallow. More accessible derivation can be found e.g. in [5, p. 44].

ematically the simplest one - they *postulate* that spinning particles should follow autoparallels of the connection with torsion. It is this part of Shipov's "Theory of Physical Vacuum" [6], as well as its applications, that I will discuss below. In my discussion I will avoid discussing interpretational problems, that is how to interpret a given mathematical formalism when applying it to physics, in particular to the experimental results. I will concentrate on mathematical problems, inconsistencies and errors alone. In my opinion making speculations about physics while relying on faulty mathematics is, at least, unhealthy. Therefore the mathematics has to be fixed first of all. I will point out the problems that caught my eye with the hope that it may help the author to fix these and similar issues in the future. I will start with problems of a lower rank, in section II, and only then move to problems of a more serious nature, in section III.

As it was in Part I, I will refer to Ref. [6] as "The Book" and to the author as "Author".

II. Problems with "Equations of geodesics of A_4 spaces"

Let me first of all point out a certain problem with the terminology. The Author gives to Ch. 5.7 of Ref. [6] the title "Equations of geodesics of A_4 spaces". It is evident from The Book that A_4 stands for spacetime manifold endowed with teleparallelism, that is with an affine connection of zero curvature but with torsion. While defining A_4 [6, p. 11] we are referred, in particular, to Schouten's works. Yet the Author, apparently, did not study Schouten's monograph "Ricci Calculus" carefully enough. Otherwise he would know that Schouten defines A_n as a space endowed with nonzero curvature, but with zero torsion [7, p. 126], [8, p. 87], just the converse of how it is presented in the Book. While the issue is not important, yet it deserves mentioning, otherwise some readers, (who did study Schouten), could be easily confused.

As it is postulated in Ref. [9], the motion center of mass of a 4-D gyroscope is governed by the equations of autoparallels (called also geodesics) of the connection Δ[3]

$$\frac{d^2x^i}{ds^2} + \Delta^i{}_{jk}\frac{dx^j}{ds}\frac{dx^k}{ds} = 0. \tag{II.1}$$

The reason for such a postulate are not given. Probably one of the reasons is that these equations are the first ones that come to mind when dealing with any affine connection. Such a reason while good for mathematicians is not always good for physicists. There is a good historical example in the beginnings of the general theory of relativity. Einstein was learning Riemannian geometry from Marcel Grossmann - a mathematician. The first thing that came to the mind of the mathematician was using the Ricci tensor for the left side of the field equations describing the coupling between geometry and matter. But it was wrong! When Einstein realized the failure, he contemplated abandoning the geometrical approach completely.[4] It took a couple more years before another mathematician, David Hilbert, who understood the importance of conservation laws and invariance principles, discovered the right way of getting the gravitational field equations - through the principle of minimal action. The solution was not that complicated - what was needed was adding another term to the Ricci tensor, to form what was later on called the Einstein tensor.

Evidently the Author understood the importance of some kind of a variational principle, at least for geodesic equations. In Ref. [9] he writes explicitly that he was able to obtain autoparallels equations (II.1) from a variational principle in the Book. Indeed in Ch. 5.7 there are two pages of formal manipulations that look like support for this claim. In fact these manipulations are almost identical to those that appeared in a paper by Fiziev in Kleinert [11], though no reference to this paper is given in the book.[5] The geometrical idea behind these manipulations is very simple. Given an affine connection, any path through a given point can be developed into the tangent space through this point, and the path is an autoparallel if, and only if, its tangent space development is a segment of a straight line (cf. e.g. [12, Ch. III.4], [13, Ch. 2.3.50]. Straight lines can be easily obtained from

[3] For details of the geometry and notation conventions cf. Part I.

[4] Interesting details can be found in Ref. [10].

[5] It is hard to tell who first came with this idea.

variational principle - but in the tangent space, not in the spacetime manifold itself. This is the essence of the method used by Fiziev and Kleinert and by the Author. Yet Kleinert and Peltzer [14, p. 1443] realize that in this way we do not really have a bona fide variational principle with variations vanishing at the end points of spacetime trajectories. Thus claims that autoparallel equations can be obtained from a variational principle should be taken with a grain of salt.[6]

In Ref. [16, p. 57] T. Lakomina and R. Polishchuk authoritatively state that every equation for a physical field arises from the principle of extremal action. This statement is evidently false. This is not the way Maxwell equations came to life; this is not the way how Schrodinger's or Dirac's equation have been derived. While it is true that afterwards it was possible to find action principles for these equations, there is no a priori reason why the action principle should be considered as mandatory. It is a good guiding principle, but it should not be repeated as a mantra. The proof of the pudding is in eating it; similarly with the "Theory of Physical Vacuum", the proof of the theory is in checking how well it deals with qualitative explanations and quantitative predictions of the result of experiments. Therefore, in the next section, I will discuss the application of the Author's ideas to the so called $4-D$ gyroscope ("inertioid").

III. Mathematical errors and contradictions in "Descartesian Mechanics: The Fourth Generalization of Newton's Mechanics"

According to the Book the center of mass of a free gyroscope follows an autoparallel of the connection. At the same time its axes preserve the constant orientation with respect to the autoparallel orthonormal tetrad e^i_a. In Ref. [9] , and also in several other publications dealing with "Descartesian mechanics" and "4-D gyroscope" the author develops applications of the autoparallel geometry with torsion, together with its postulated equations of motion to experiments with some rather special gyroscopic devices. As it is described in [9], [17] typically such a device ("inertioid") *consists of three connected masses, two of which (masses m) rotate synchronously in different direction in the spatial angle $\phi(t)$ around axis O, set on the central mass M.*. The device contains also a control mechanism. It is such a device that the Author attempt to model with the mathematics of autoparallel geometry. Let me first describe the error in the final formula for the center of mass acceleration, the formula that was used by V. Zhigalov in his comparison [18, Eq. (1)] (see also Ref. [19]) of experimental results with the theory.

The motion of the device is essentially planar, therefore it is being modeled with only two degrees of freedom: the angle ϕ and the center of mass position x_c. The

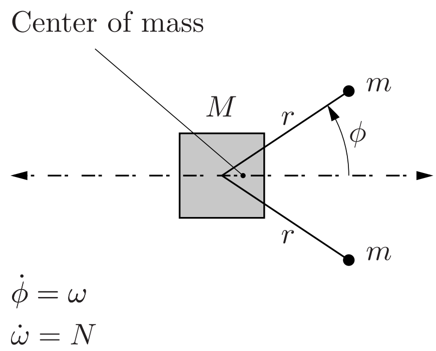

Figure 1. Schematic diagram of the inertioid.

coordinates used are

$$x_0 = ct, x_1 = x_c, x_2 = r\phi.$$

The metric that is postulated is of the form:

$$[g_{ij}] = \begin{bmatrix} 1 - 2k^2r^2U(\phi)/c^2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -k^2(1-k^2\sin^2\phi) \end{bmatrix}, \qquad ((94))$$

where $k = 2m/(2m+M)$, and the "potential"

$$U(\phi) = \int_{\phi_0}^{\phi} N, \qquad ((95))$$

is crated by angular acceleration N.

[6] Of course one can always obtain what one wants to obtain by introducing by hand artificial constraints and Lagrange multipliers, as, for instance, in [15], where the essence is hidden behind complicated formulas and mathematical formalism.

The nonzero components of the totally antisymmetric contorsion are chosen as

$$T^1_{20} = -T^1_{02} = \frac{k^2\Phi}{2c},$$
$$T^2_{10} = -T^2_{01} = \frac{\Phi}{2c(1-k^2\sin^2\phi)}. \qquad ((99))$$

Remark 3.1: In Ref. [9] there is an evident contradiction between Eqs. ((98)) and ((99)). The coefficients and signs of ((99)) do not agree with those of ((98)), as for the totally antisymmetric torsion we should have $T = -\Omega$. Therefore I have corrected ((99)) in such a way as to obtain the closest reproduction of the final formula ((107))

Remark 3.2: Evidently the "metric" ((94)) is **not** a space-time metric. Writing $U(\phi)$ is misleading, since this term is not a function of ϕ but of $\omega = \dot{\phi}$. Therefore ((94)) should be considered as a part of a Finslerian metric, that is of a metric on the tangent bundle (see e.g. [20] and references therein). Since nowhere the Author defines a complete Finslerian geometry on the tangent bundle, the whole following discussion lacks a sound mathematical support. Nevertheless I will try to follow the Author's heuristic reasoning.

The condition of teleparallelism (vanishing of the curvature of $\Delta = \Gamma + T$),

$$R^i{}_{jkm} + P^i{}_{jkm} = 0, \qquad (III.1)$$

where

$$P^i{}_{jkm} = 2\nabla_{[k}T^i_{|j|m]} + 2T^i_{s[k}T^s_{|j|m}. \qquad ((5.130))$$

That is

$$P^i{}_{jkm} = \nabla_k T^i_{jm} - \nabla_m T^i_{jk} + T^i_{sk}T^s_{jm} - T^i_{sm}T^s_{jk}. \qquad (III.2)$$

Contracting i, k

$$R_{jm} + P_{jm} = 0, \qquad (III.3)$$

$$P_{jm} = \nabla_i T^i_{jm} - \nabla_m T^i_{ji} + T^i_{si}T^s_{jm} - T^i_{sm}T^s_{ji}. \qquad (III.4)$$

For totally antisymmetric T we have $T^i_{ji} = 0$, therefore, in this case

$$P_{jm} = \nabla_i T^i_{jm} - T^i_{sm}T^s_{ji}. \qquad (III.5)$$

Contracting with g^{jm}, and taking into account the fact that for totally antisymmetric T we have $T^i_{jm} = -T^i_{mj}$

$$R + P = 0, \qquad (III.6)$$
$$P = -g^{jm}T^i_{sm}T^s_{ji}.$$

From these last two equations the Author deduces the following form of Φ

$$\Phi = 2\sqrt{\frac{N\sin\phi\cos\phi}{1-k^2\sin^2\phi} + \frac{N_\phi}{k^2}}. \qquad ((107))$$

Yet his calculations contain an error - one term is missing. The correct formula deduced from Eqs. III.6 reads

$$\Phi = 2\sqrt{\frac{N\sin\phi\cos\phi}{1-k^2\sin^2\phi} + \frac{N_\phi}{k^2} + \frac{N^2r^2}{c^2 - 2k^2r^2U(\phi)}}. \qquad (III.7)$$

In order to get an idea about the shape of the term with $U(\phi)$, we notice that

$$N = \dot{\omega} = \frac{d\omega}{d\phi}\frac{d\phi}{dt} = \frac{d\omega}{d\phi}\omega = \frac{1}{2}\frac{d\omega^2}{d\phi}. \qquad (III.8)$$

Suppose that on a certain time interval ω depends on t through ϕ:

$$\omega(t) = \omega(\phi(t)). \qquad (III.9)$$

Then $N = \dot{\omega}$, $N_\phi = \frac{dN}{d\phi} = \frac{dN}{dt}\frac{dt}{d\phi} = \frac{\ddot{\omega}}{\omega}$. Therefore

$$U(\phi) = \int_{\phi_0}^{\phi} N = \frac{1}{2}(\omega^2 - \omega_0^2). \qquad (III.10)$$

The wrong formula ((107)) leads to the wrong formula for the center of mass acceleration analyzed in [18]:

$$a_{c.m.} = 2B\omega\sqrt{\frac{\dot{\omega}\sin\phi\cos\phi}{1-A\sin^2\phi} + \frac{\ddot{\omega}}{A\omega}}, \qquad (III.11)$$

$$A = \frac{2m}{2m+M}, \quad B = rA. \qquad (III.12)$$

The result of the analysis in [18] is negative: the formula (III.11) cannot explain the available experimental data. But now, knowing that there is a missing term in the formula obtained by the Author, the question arises if the corrected formula could do better than the original, wrong one? From the form of the missing term it is seen that this term vanishes in the limit $c \to \infty$, therefore taking it into account would not make much difference.

A. *Internal inconsistency of the whole approach*

In fact, not only the formula ((107)) of Ref. [9] is wrong, but the whole derivation of this formula is mathematically inconsistent. The formula ((107)) is derived from Eqs. (III.6). But (III.6) is obtained from (III.5) (Eq. ((103)) in [9]). Yet the Ansatz ((99)) used by the Author contradicts (III.5). For instance $R_{10} = 0$, but

$$P_{10} = \frac{k^2 \Phi \sin\phi}{4cr\left(k^2 \sin^2\phi - 1\right)^2}. \qquad \text{(III.13)}$$

It is clear that $R_{10} + P_{10} \neq 0$. Therefore *the fundamental requirement for the autoparallel geometry is violated: the total curvature is not zero*.

Why did the Author overlook this fact? A reason for this negligence can be found in a misleading sentence right after Eq. ((99)) in Ref. [9], where the Author states that "the Ricci torsion in the Cartan's structural equations of the geometry A_4 does not depend on metric." Eq. (III.3) *directly contradicts this statement*. While R_{jm} depends on the metric alone, and P_{jm} depends on the torsion, the sum $R_{jm} + P_{jm}$ must be zero! Therefore one is not independent of the other. While torsion can indeed be set arbitrarily in a general Einstein-Cartan theory, it is not so when the condition of zero curvature is imposed, as it is done, from the very beginning in the Book and in the following papers.[7]

IV. Some additional observations

While the Author speaks about "The Fourth Generalization of Newton's Mechanics", the 2005 NASA Report on "Advanced Energetics for Aeronautical Applications" [21] discusses "The Fourth Law of Motion". To quote from the introduction in section 5.2.1:

> This report presents an introduction and overview to a topic with many fundamentally far reaching implications and applications. The Fourth Law of Motion is so-named because it has been applied to measurable physical phenomena that are not accurately explained by Sir Issac Newton's classical Three Laws of Motion. The application of the Fourth Law of Motion could potentially lead to a better understanding of many topics, including but not limited to, transient phenomena, shock waves, thermodynamics, some of the approaches used to attempt to access ZPE, and some of the approaches used to attempt to construct a propellantless (i.e., reactionless) propulsion system.

This part of the NASA report is concerned with what is known as "Davis' mechanics", where Newton's equations of motion are extended to contain the third derivative with respect to time. The so called "Dean' drive" (cf. Fig. 2)[8] bears certain similarities with "inertioids" (both have counter-rotating masses).

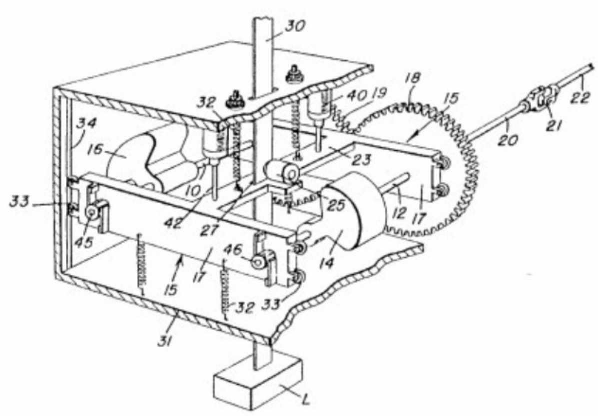

Figure 2. Schematic diagram of the Dean drive.

The interested reader can find more details in Ref. [23]. While there are some speculations that the observed phenomena may have something to do with "Kozyrev's causal mechanics", N. A. Kozyrev himself dismissed such associations [24]. It is interesting that while Davis' mechanics explicitly discuss the third time derivative, such a derivative appears implicitly in the equations analyzed by V. Zhigalov [18], where he notices that it is extremely difficult to measure experimentally this quantity. Perhaps the similarity between Davis' ideas of "The Fourth Law of Motion" and "The Fourth Generalization of Newton's

[7] Though it should be noted that the Ansatz is consistent in the non-relativistic limit $c \to \infty$. Mathematica notebook containing all relevant calculations can be found at URL http://www.arkadiusz-jadczyk.eu/docs/shipov_gyro4.nb

[8] More details at http://en.wikipedia.org/wiki/Dean_drive. For a classical mechanical engineering analysis of similar devices cf. Ref. [22] and references therein.

Mechanics" is just coincidental. But it is also possible that theoretical physics has yet to discover something important about higher derivatives in the mathematical formulation of the fundamental laws of physics.

V. Conclusions

As I have already mentioned in Part I, it would be logically wrong to draw the conclusions that mathematical errors and contradictions found in one part of the work discredit the whole idea of "teleparallel geometries" and the attempts at construction of some kind of a "unified field theory" based on similar ideas. However the particular implementation of these ideas as presented in The Book and in other papers of the Author is mathematically faulty, therefore it cannot serve as a basis for a healthy physical theory. To quote from Part I:

> Modern theoretical physics requires advanced mathematics, and anyone using such mathematical tools should, first of all, have a clear understanding of the meaning of mathematical operations and formulas. Otherwise confusion and misinterpretation will prevail.

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